**Roll No. ..........................** **Total Pages : 04**

**BT-I/D-21** **41001**

MATHEMATICS

MATH-101-E

Time : Three Hours] [Maximum Marks : 100

**Note** : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

**Unit I**

**1.** (a) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the Folium $x^3 + y^3 = 3axy$.

(b) Find the asymptotes of the curve $x^2y - xy^2 + xy + y^2 + x - y = 0$ and show that the cut the curve again in three points which lie on the line $x + y = 0$.

**2.** (a) Prove that (by using series) :

$$\frac{x}{2}\left(\frac{e^x + 1}{e^x - 1}\right) = 1 + \frac{1}{6}.\frac{x^2}{\angle 2} - \frac{1}{30}.\frac{x^4}{\angle 4} + .......,$$

where $\angle$ denotes the factorial notation.

(b) Trace the curve :

$$y^2(2a-x)=x^3$$

## Unit II

**3.** (a) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$, show that :

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$$

(b) If $\sin u=\frac{x^2y^2}{x+y}$, prove that :

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3\tan u$$

**4.** (a) If $u=x+3y^2-z^3$, $v=4x^2yz$, $w=2z^2-xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0).

(b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

## Unit III

**5.** (a) Change the order of integration in $\mathrm{I}=\int_0^1\int_{x^2}^{2-x}xy\,dx\,dy$ and hence evaluate the same.

(b) Evaluate :

$$\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$$

**6.** (a) Prove that :

$$\beta(m, n) = \frac{\overline{)m}\,\overline{)n}}{\overline{)m+n}}$$

(b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing to polar coordiantes. Hence show that :

$$\int_0^\infty e^{-x^2} = \sqrt{\frac{\pi}{2}}$$

## Unit IV

**7.** (a) If A = $5t^2$I + $t$J − $t^3$K, B = sin$t$I − cos$t$J, find :

(i) $\frac{d}{dt}(\mathrm{A}.\,\mathbf{B})$ (ii) $\frac{d}{dt}(\mathrm{A} \times \mathbf{B})$

(b) Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point (2, −1, 1) in the direction of vector I + 2J + 2K.

**8.** (a) Show that :

$$\nabla^2 (r^n) = n(n+1) r^{n-2}$$

(b) Apply Green's theorem to evaluate :

$$\int_C \left[ \left(2x^2 - y^2\right) dx + \left(x^2 + y^2\right) dy \right],$$

where C is the boundary of the area enclosed by the $x$-axis and the upper-half of the circle $x^2 + y^2 = a^2$.